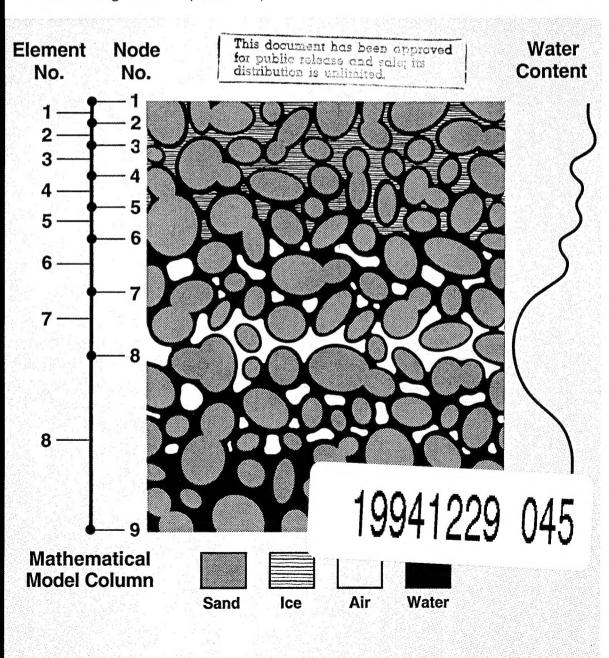




Soil Moisture Prediction During Freeze and Thaw Using a Coupled Heat and Moisture Flow Model

Susan R. Bigl and Sally A. Shoop

November 1994



Abstract

A coupled heat flow and moisture flow model (FROSTB) was used to simulate large scale freeze—thaw experiments to assess its ability to predict soil moisture conditions during freeze and thaw. The experimental data consists of temperature and soil moisture profiles through freeze—thaw cycles of a 1-m layer of frost-susceptible silty sand over roughly 2 m of gravely sand. Two experimental conditions were modeled: 1) where the soil moisture was lower than specific retention (less than 12% by weight) and no water table was present (dry case) and 2) where the soil was fairly wet and the water table was approximately 1 m deep (wet case). During freezing, FROSTB tends to predict ice contents higher than those observed, which causes the simulated soil column to thaw slower. During thawing the predicted moisture contents in the thawed soil were close to the measured values for the wet case but were always higher than the measured moisture contents for the dry case. Possible reasons for the discrepancy are discussed.

Cover: Moisture distribution (right) during freezing of an unsaturated soil, showing the frozen layer on top underlain by a drier zone and then the saturated soil (water table). The moisture migration during freeze—thaw is modeled by dividing it into discrete nodes and elements with assigned material properties (left).

For conversion of SI metric units to U.S./British customary units of measurement consult *Standard Practice for Use of the International System of Units (SI)*, ASTM Standard E380-89a, published by the American Society for Testing and Materials, 1916 Race St., Philadelphia, Pa. 19103.

CRREL Report 94-11

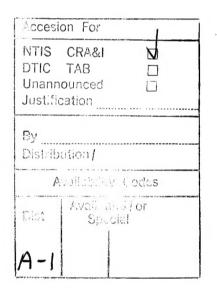


Cold Regions Research & Engineering Laboratory

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PREFACE

This report was prepared by Susan R. Bigl, Research Physical Scientist, Civil and Geotechnical Engineering Research Branch; and Sally A. Shoop, Research Civil Engineer, Applied Research Branch, Experimental Engineering Division, U.S. Army Cold Regions Research and Engineering Laboratory.

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NOMENCLATURE

A_w , α	Gardner fit coefficients for soil moisture	\boldsymbol{x}	coordinate (positive downward)
A_K , β	Gardner fit coefficients for hydraulic conductivity	ϕ_{w}	liquid water flow potential; three-phase system
E	phenomenological calibration factor for partly frozen soil	ф _{аw}	flow potential; air–water–soil system $(\phi_{aw} = h_p)$
8	gravitational constant	ϕ_{iw}	flow potential; ice-water-soil system
h	total hydraulic head ($h = h_p + h_e$)	θ_{a}	volumetric air content
h_{e}	elevation head $(h_e = -x)$	θ_{i}	volumetric ice content
h_{p}	pressure head $(h_p = u/\gamma_w)$	θ_n	volumetric unfrozen water content factor
$k_{ m s}$	saturated hydraulic conductivity (unfrozen soil)	$\theta_{\rm o}$	for frozen soil porosity
$K_{ m F}$	hydraulic conductivity of partly frozen	θ_{s}	volumetric segregated ice content
	soil	θ_{u}	volumetric water content (unfrozen)
K_{H}	hydraulic conductivity (unfrozen soil)	$\gamma_{\rm w}$	unit weight of water $(\gamma_w = g\rho_w)$
t	time	ρ_{i}	density of ice
и	pore fluid pressure	ρ_{w}	density of water
$W_{ m grav}$	gravimetric water content		

Soil Moisture Prediction During Freeze and Thaw Using a Coupled Heat and Moisture Flow Model

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INTRODUCTION

This study assessed the accuracy of the FROSTB freeze-thaw model for predicting soil moisture during freeze and thaw. The results of simulations were compared to a data set compiled during a vehicle mobility project conducted in CRREL's Frost Effects Research Facility (FERF), a building in which full-scale field tests are conducted with soils up to 3.6 m (12 ft) deep (Shoop et al. 1991). FROSTB was used to simulate two freeze-thaw cycles: one with relatively dry conditions and one in which a water table was established at 0.9-1.2 m (3-4 ft) below the surface. The simulation predictions were compared with the measured soil temperature, soil moisture and frost heave. This study emphasized soil moisture predictions, which were compared to several data sets: moisture tension, moisture contents from frozen core and thawed soil samples, and water table measurements at the standpipes.

BACKGROUND ON THE MODEL

FROSTB is a one-dimensional coupled heat flow and moisture flow model that computes frost heave and thaw settlement of a pavement or soil profile with time. It also calculates soil temperature, moisture stress, water content, ice content and density through the depth of the profile at each time increment. Berg et al. (1980) originally developed FROSTB in a cooperative study funded by the Corps of Engineers, the Federal Highway Administration and the Federal Aviation Administration. Additional details beyond those described here are given in Guymon et al. (1993). The model assumes one-dimensional vertical heat and moisture flux and is based on a numerical solution technique termed the nodal domain integration method. This method allows the same computer program to be used to solve a problem by either the finite element

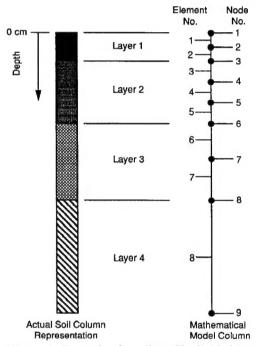


Figure 1. Example of a soil profile divided into nodes and finite elements.

method, the integrated finite difference method or any other mass lumping numerical method.

Figure 1 shows how FROSTB uses nodes, which are exact points, to divide the column of material into horizontal elements. Material properties are assigned to the elements.

The program was developed for solving problems of seasonal freezing and thawing of nonplastic soils and is based on the following primary assumptions, with additional assumptions reported in Guymon et al. (1993):

- Darcy's law applies to moisture movement in both saturated and unsaturated conditions.
- The porous media are nondeformable as far as moisture flux is concerned; i.e. consolidation is negligible.

- All processes are single valued; i.e. hysteresis is not present in relationships such as the soilwater characteristic curve.
- Water flux is primarily as a liquid; i.e. vapor flux is negligible.

The governing equation used in FROSTB to describe soil moisture flow is derived by substituting the extended Darcy moisture-flow law into the one-dimensional continuity equation for an incompressible fluid flowing through porous media:

$$\frac{\partial}{\partial x} \left[K_{\rm H} \frac{\partial h}{\partial x} \right] = \frac{\partial \theta_{\rm u}}{\partial t} + \frac{\rho_{\rm i}}{\rho_{\rm w}} \frac{\partial \theta_{\rm i}}{\partial t} \tag{1}$$

where K_H = unsaturated hydraulic conductivity (permeability) (cm/hr)

h = total hydraulic head (cm of water)

x = depth (cm)

 θ_{u} = volumetric unfrozen water content (%)

 ρ_i = density of ice (g/cm³)

 $\rho_{\rm w}$ = density of water (g/cm³)

 θ_i = volumetric ice content (%)

t = time (hr).

The total hydraulic head h equals the sum of the pore pressure head ($h_p = u/\gamma_W$, where u is the pore water pressure and γ_W is the unit weight of water) plus the elevation head ($h_e = -x$, where x is measured vertically downward). The ice sink term, $\rho_i \partial \theta_i/\rho_W \partial t$, exists only for a freezing or thawing zone, and in these zones eq 1 is coupled to the heat transport equation. The ice sink term assumes that θ_i is a continuous function of time.

In FROSTB the soil water characteristics are represented using an equation in the form of Gardner's (1958) function:

$$\theta_{\rm u} = \frac{\theta_{\rm o}}{A_{\rm w} \left| h_{\rm p} \right|^{\alpha} + 1} \tag{2}$$

where θ_0 = soil porosity (%)

 h_p = pore pressure head (cm of water)

 $A_{\rm w}$ = Gardner's multiplier for the moisture characteristics

α = Gardner's exponent for the moisture characteristics.

For each soil to be modeled, point values of θ_u and h_p are determined in a laboratory moisture retention test (Ingersoll 1981). Equation 2 is then fit to the data using a least-squares approach to determine the best-fit parameters A_w and α .

Unsaturated hydraulic conductivity is also approximated in FROSTB using an equation from Gardner:

$$K_{\rm H} = \frac{k_{\rm s}}{A_{\rm K} \left| h_{\rm p} \right|^{\beta} + 1} \tag{3}$$

where K_H = unsaturated hydraulic conductivity (cm/hr)

 k_s = saturated hydraulic conductivity (cm/hr)

 A_{K} = Gardner's multiplier for hydraulic conductivity

 β = Gardner's exponent for hydraulic conductivity.

Point values of $K_{\rm H}$ and $h_{\rm p}$ for each soil are determined in the laboratory by an unsaturated hydraulic conductivity test (Ingersoll 1981), and eq 3 is fit to the data using a least-squares approach to determine the best-fit parameters $A_{\rm K}$ and β .

Within the partially frozen zone, FROSTB reduces the unsaturated hydraulic conductivity using an empirical constant, termed the *E*-factor, combined with the ice content according to the following equation:

$$K_{\rm F} = K_{\rm H} | (h_{\rm p}) | \cdot 10^{-E\theta_{\rm i}} , E\theta_{\rm i} \ge 0$$
 (4)

where $K_{\rm F}$ is the adjusted hydraulic conductivity in a partially frozen element (cm/hr) and E is the empirical constant (dimensionless). The E-factor can be set by the user or determined within the FROSTB program using an empirically derived equation based on the saturated hydraulic conductivity, $k_{\rm S}$, in centimeters per hour:

$$E = \frac{5}{4} (k_{\rm s} - 3)^2 + 6. \tag{5}$$

An example of the function predictions relative to calibration points is shown in Figure 2. Additional discussion of the *E*-factor is given in Guymon et al. (1993).

Frost heave is estimated from the total amount of ice segregation in the frozen zone by

$$\theta_{s} = \theta_{i} - (\theta_{o} - \theta_{n}) \tag{6}$$

where θ_s is the volumetric segregated ice content (%) and θ_n is the minimum volumetric unfrozen water content (%). If θ_s is greater than 0, ice segregation has occurred and the frost heave is computed by multiplying θ_s by the element length. The θ_n parameter establishes the pore water stress at the freezing front for the solution of the moisture transport equation. In this study, θ_n was obtained by assuming a moisture tension of -800 cm of water

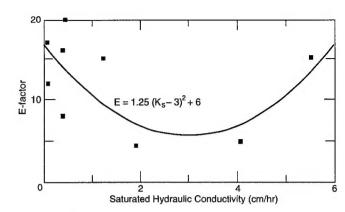


Figure 2. E-factor function compared with calibration data points. (Data from Guymon, pers. comm.)

and solving eq 2. Thaw consolidation from ice melting is the reverse process of that described above for ice segregation. Upon thawing, water in excess of the porosity is treated as a source, forcing upward drainage. It is assumed that upon reaching the surface, the water drains away laterally.

To conduct the calculations described above, FROSTB requires the following input for each material:

- Gardner's coefficients for soil moisture characteristics;
- Gardner's coefficients for hydraulic conductivity characteristics;
- Porosity and density of the soil;
- Thermal conductivity and volumetric heat capacity of the dry soil; and
- The E-factor.

FROSTB also requires the following input for initial and boundary conditions:

- · Element lengths;
- Upper- and lower-boundary pore water pressures with time;
- Upper- and lower-boundary temperatures with time:
- Initial temperature, pore pressure and ice content distributions with depth; and
- Surcharge pressure.

EXPERIMENTAL VALIDATION

The FERF building is temperature controlled so that full-scale freeze-thaw tests can be conducted year-round. The building is divided into test cells measuring approximately 6.1 m (20 ft) square and 3.7 m (12 ft) deep. The mobility testing was conducted in six of the northernmost cells. Freeze events were created by placing refrigerated panels on the soil surface. When frost penetrated the desired amount, the panels were removed while the ambient temperature in the building was below freezing. The building temperature remained below freezing while a level survey was conducted to determine the amount of frost heave, and frozen cores were drilled to retrieve samples for measuring ice contents. Then either the building was heated or warm outside air was allowed to enter the building to provide heat for thawing the soil from the surface. During thaw, samples of the thawed soil were extracted and tested for density and moisture content.

The two soils in the test cells, Lebanon sand and Pompey Pit sand, have size gradations as shown in Figure 3. Both soils are classified as sands with fines (SM) in the Unified Soil Classification System. Their

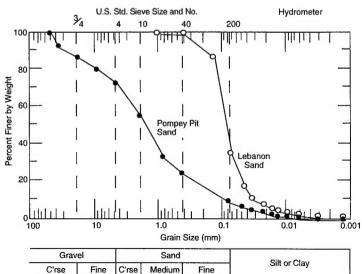


Figure 3. Size gradation of materials in the test sections.

Table 1. Layer conditions in FERF mobility test cells.

		Dept	h range	Dry density	
Material	Condition	(in.)	(cm)	(lb/ft³)	(Mg/m^3)
Tested conditions					
Lebanon sand	dry	0-6	0-15.2	98.6	1.551
	wet	0-6	0 - 15.2	101.1	1.620
Installed conditions					
Lebanon sand		6 - 18	15.2-45.7	104.8	1.678
Lebanon sand		18 - 42	45.7-106.7	106.0	1.697
Pompey Pit sand		42-138	106.7 - 350.5	123.5	1.978

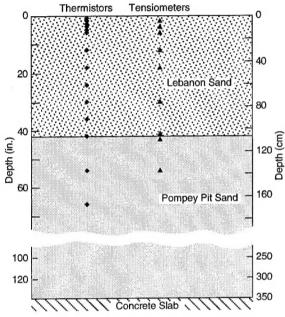
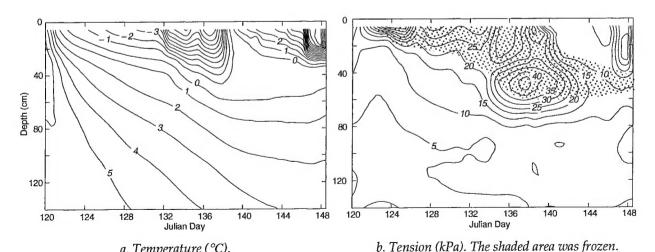


Figure 4. Material depths and instrumentation locations.

specific gravities are 2.71 for Lebanon sand and 2.74 for Pompey Pit sand. They were layered with 1.07 m (42 in.) of Lebanon sand over 2.44 m (8 ft) of Pompey Pit sand, which rests on an underlying concrete slab. During placement the Pompey Pit sand had a relatively constant density and moisture content (Table 1). The Lebanon sand had variable density in three layers: the bottom two layers resulting from the placement preparations, and the top 15.2-cm (6-in.) layer from preparations (watering or drying, tilling and compaction) for a particular test.

Three variables were measured on a regular basis to monitor the condition of the test cells. Thermistors measuring temperatures were positioned at 2.54-cm (1-in.) intervals down to a depth of 15.24 cm (6 in.), at 15.24-cm (6-in.) intervals down to 106.68 cm (42 in.) and at 30.48-cm (12-in.) spacing down to 167.64 cm (66 in.) (Fig. 4). The thermistors were polled by an automatic datalogger, and readings were recorded at 2-hour intervals. However, for the wet case there were a few periods when data were not recorded. Tensiometers, positioned as shown in Figure 4, were read manually once per day throughout the test period to measure pore water tensions.



a. Temperature (°C). b. Tension (kPa Figure 5. Measured data for the dry freeze event.

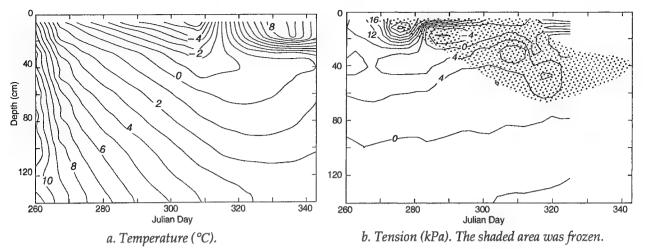


Figure 6. Measured data for the wet freeze event.

Data from frozen tensiometers were disregarded, since they do not work properly when frozen. When a water table was established in the cells, daily water levels were measured at standpipes at the edge of the test cells.

The two freeze events simulated with the FROSTB program were characterized by a relatively dry condition (no water table) and a wet condition (with a water table). The dry event involved a cross section that had the installed density conditions indicated in Table 1. Panels were placed on the soil surface from 29 April to 17 May 1988 (Julian day 120-138). In the second (wet) event, the panels were on the section from 19 September to 6 November 1988 (JD 263-311). The position of the water table in the wet event is a little unclear: the tensiometers indicated an average depth of 94 cm (37 in.); at the same time, elevation measurements in the standpipe indicated a water table depth of 119 cm (47 in.). The discrepancy may be because of an uneven surface at the test basin edge where the standpipes were located and variations in the water table depth as water was added. The temperature and tension data collected during these dry and wet freeze events are shown, respectively, in Figures 5 and 6 (relationships between soil tension and water content are shown in Fig. 7).

Final heave was measured by conducting level surveys at marked points on the soil surface and comparing them with like measurements prior to the freeze event. The moisture content of the frozen cores was determined by sectioning the core into 2.54-cm (1-in.) pieces and determining gravimetric water content using standard procedures. The gravimetric water content of thawed soil samples were also determined. Heave and water content data will be presented in comparison with the predicted values.

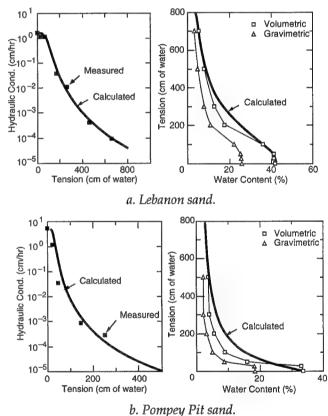


Figure 7. Modeled and measured values of hydraulic conductivity and moisture retention.

FROSTB SIMULATIONS—GENERAL INPUT

Soil parameters

The various soil physical and hydraulic characteristics used to simulate material properties are shown in Table 2. Comparisons of the measured hydraulic data relative to values predicted by the Gardner's equations are given in Figure 7. Thermal properties used in the simulations are shown in Table 3. The

Table 2. Physical and hydraulic properties input to FROSTB simulations.

			Lebanon sand		
Property		Layer 1 (0–15 cm)	Layer 2 (15–46 cm)	Layer 3 (46–107 cm)	Pompey Pit sand
Soil density (g/cm ³)		1.551*	1.678	1.697	1.978
Soil porosity (cm ³ /cm ³)		0.419	0.419	0.419	0.336
Soil water characteristics:	A_{w}	1.962×10^{-5}	1.962×10^{-5}	1.962×10^{-5}	3.7116×10^{-3}
	α	1.975	1.975	1.975	1.268
Min, unfrozen water cont. (cm ³ /cm ³)		0.03602	0.03602	0.03602	0.01787
Saturated hydraulic cond. (cm/hr)		1.6	1.6	1.6	5.5
Permeability characteristics:	A_{K}	1.590×10^{-9}	1.590×10^{-9}	1.590×10^{-9}	2.875×10^{-5}
	β	4.623	4.623	4,623	3.806

^{* 1.551} g/cm³ for dry case, 1.620 g/cm³ for wet case.

value of 33.5 cal/cm hr °C was used for soil solids in all cases except one (wet case 6b), which used the value 17.0 cal/cm hr °C.

Table 3. Material thermal properties.

Material	Specific heat (cal/g°C)	Thermal conductivity (cal/cm hr °C)
Water	1.00	5.0
Ice	0.55	18.0
Soil particles	0.2	33.5*

^{*} A value of 17.0 was used for wet case 6b.

Initial and boundary conditions

The soil column was simulated with the upper boundary at the position of the uppermost functioning thermistor and the lower boundary at the bottom of the granular material, which was located at the upper surface of a concrete slab beneath the test cells. The total thickness of material was 350.52 cm (11.5 ft). The depth from the soil surface to the uppermost thermistor was 5.08 cm (2 in.) for Cell 10, the dry case, and 2.54 cm (1 in.) for Cell 11, the wet case. In all cases the soil column was simulated with 99 elements: 2-cm-long elements between the surface and 105 cm, 4-cm elements between 105 and 225 cm, 5-cm elements between 225 and 245 cm, and 10-cm elements from 245 cm to the bottom of the profile.

The upper-boundary pore water pressure was chosen to be computer generated, as follows. When the profile is completely thawed and downward vertical drainage occurs, the surface pore water boundary condition is modeled by

$$\frac{\partial h}{\partial x} = 0 \tag{7}$$

which means that the velocity flux across this boundary is zero. The upper-boundary condition is set to 0 cm of water when the upper-boundary temperature is above 0°C and frozen regions remain in the column. When the surface temperature is below

0°C, a specified constant upper-boundary pore pressure (–300 cm of water) is used.

In general the lower-boundary pore pressure condition of FROSTB is set by specifying discrete pore water pressures (tensions) that relate to the water table elevation at times when these conditions occur. At intermediate times, lower-boundary pore water pressures are linearly interpolated. For all the cases in this study, we set a constant lower-boundary pore pressure, which produced a constant water-table depth throughout the simulation. Specific cases will be discussed in a later section.

Input for the upper-boundary temperature condition consists of a set of specified times and temperatures that are implemented as step changes. The temperature input for this study was the measured data at the upper thermistor for 6-hour time increments. For the wet case there were periods with missing data, and these were estimated using a linear interpolation between the closest available data points.

When air temperatures are the input values for upper-boundary temperatures, FROSTB adjusts the air temperature values to represent the soil—air interface temperatures using a procedure similar to the U.S. Army Corps of Engineers' *n*-factor approach for seasonal freezing indices (Department of the Army 1966). An *n*-factor is defined as the ratio of the surface index to the air index (normally measured at 2 m above the surface), separately calculated for the full freeze or thaw season. Because the input values for this study were actual soil measurements, the *n*-factor used was 1.0.

Bottom-boundary temperature conditions consist of a set of times and temperatures. Temperatures are linearly interpolated at intermediate times. A constant bottom temperature was specified in this study at a value equal to the estimated value at the concrete interface. The value was calculated by extrapolating the gradient between the bottom two thermistor measurements at the start of the simulation.

The initial conditions required to be set for the FROSTB program are the temperature, the pore water pressure and the ice content of each node. The initial temperature condition was set by interpolating between the measured temperatures down to the deepest measurement at 167.6 cm (66 in.) and extrapolating the gradient between the bottom two measurements down to the bottom of the cell. The initial pore water pressure was set by interpolating between the measured tensions down to the deepest measurement at 137.2 cm (54 in.), beneath which it was increased positively downwards with a gradient of 1 cm of water for each centimeter increase in depth. Simulations began before the freeze event started, so the initial volumetric ice content was set at 0.0% for all elements.

FROSTB allows for a constant surcharge (overburden) pressure to simulate the pressure acting on the top node of the modeled column. This was set in the simulations to represent the pressure of the upper 2.54 or 5.08 cm (1 or 2 in.) of soil that was above the shallowest thermistor.

The freezing point depression is a constant value that represents the temperature at which water freezes. This was set in all cases to be 0°C.

Standard procedures used in the FROSTB calcu-

lations were selected as follows: the fully implicit method was used for the moisture solution, and the Crank–Nicolson method was used for the heat transfer solution. Simulations were run with a time step of 0.2 hours, which is the time at which boundary conditions are adjusted; updates of the thermal and hydraulic properties were set to occur once per hour.

FROSTB SIMULATION RESULTS

Wet case

Four of the simulations that were run to model the wet case will be described. All cases started with the initial conditions shown in Figure 8. Table 4 summarizes the parameters adjusted for these simulations and the results.

In the first case, referred to as 1b, the lower-boundary pressure was set to match the water table depth (94 cm) indicated by the tensiometers, and the *E*-factor (8) was calculated by the program. The predicted results in this case were that the heave would be about twice that of the measured range and the frost penetration would be slightly less than measured (Fig. 9a). When the *E*-factor was manually set to a value of 24, which forced the

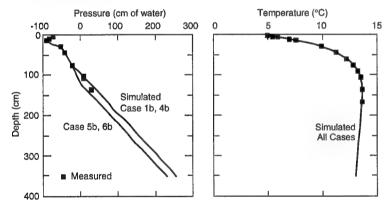


Figure 8. Initial conditions of temperature and pore water pressure (wet case, Julian Day 259) (10 cm of water = 1 kPa).

Table 4. Wet case simulation summary.
Cell 11/12; Initial temps from Julian Day 259; Upper boundary at 2.54-cm (1-in.) depth.

	v	Vater tabl			
Case	Lower pressure	(cm)	E-factor	TKSL*	Results
1b	Match data from tensiometer	94	Calc (8)	33.5	Heave higher than measured Frozen and thawed moisture high
4b	Match data from tensiometer	94	Set (24)	33.5	Heave/max frost match Frozen moisture high Thawed moisture closer
5b	Match data from standpipe	119	Set (24)	33.5	Heave/frost match Moisture high Thaw penetration quick
6b	Match data from standpipe	119	Set (24)	17.0	Thaw penetration better Max frost too shallow

^{*} Thermal conductivity of soil particles (cal/cm hr °C).

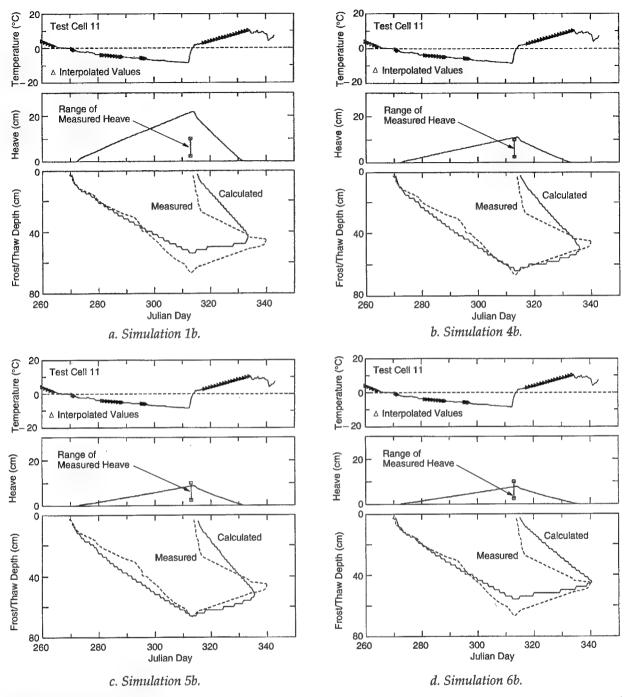


Figure 9. Applied upper-boundary temperatures, predicted frost heave and frost penetration compared to measured values for the wet case.

predicted heave to match the measured (case 4b), the maximum frost penetration also correlated well (Fig. 9b). In a third case (5b) the lower-boundary pressures were changed to set the water table at the level measured at the standpipe (119 cm). This resulted in slightly less heave and slightly more frost penetration (Fig. 9c). In all of these cases the predicted time of complete thawing is shorter than that measured. To match the thaw duration, an-

other simulation was run with a lower thermal conductivity of the soil particles (17.0 cal/cm hr°C), which caused the end-of-thaw timing to match but reduced the predicted maximum frost penetration to less than the measured amount (case 6b, Fig. 9d).

Contour plots of predicted temperatures (Fig. 10) show trends similar to the frost penetration plots and can be compared with the measured data shown in Figure 6a. In general the contour plots of

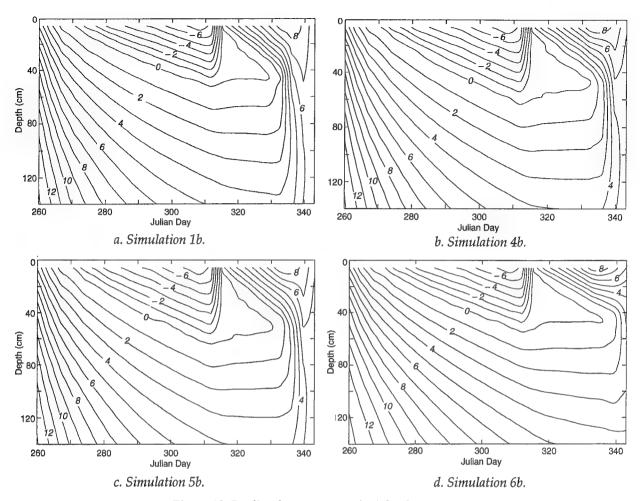


Figure 10. Predicted temperatures (°C) for the wet case.

predicted pore water pressure (Fig. 11) do not compare well with measured tension values (Fig. 6b). The predicted values at shallow depths are much higher than those measured. The depth to a tension value of 0 kPa in cases 1b and 4b do correlate, since the boundary conditions were set to force this occurrence. However, it is encouraging that the position of the measured 4-kPa tension and the predicted –5-kPa pressure contours are in similar positions.*

Moisture data from the frozen core and thawed soil samples were acquired as gravimetric water contents (W_{grav}). To compare the gravimetric data with the volumetric values of water (θ_u) and ice (θ_i) content predicted by FROSTB, the predicted values were converted to gravimetric form as follows:

$$W_{\text{grav}} = \frac{\theta_{\text{u}} + 0.9 \,\theta_{\text{i}}}{\text{dry density}}.$$
 (8)

Figure 12 compares the predicted values for all four wet cases with the measured gravimetric water content from the frozen core taken the day before thaw began. The predicted values for the frozen soil are substantially higher than the measured values.

Figure 13 compares the predicted water content with the measured data from the thawed soil samples for three situations after thaw begins. FROSTB predictions of water content for the first day of thaw (Fig. 13a) are still much higher than measured. On the second day of thaw (Fig. 13b) the predictions are much closer to the measured data at the shallowest depths. Figure 13c compares the predictions of water content for the day in which FROSTB predicts a thaw depth equivalent to that measured on the second day of thaw (Julian Day 320). On that day FROSTB predicts water contents within the thawed region similar to those measured.

Dry case

Three of the simulations that were run to model the dry case will be described. The variables adjusted for these simulations, and a short description

^{*} Note that positive tensions are equivalent to negative pore pressures.

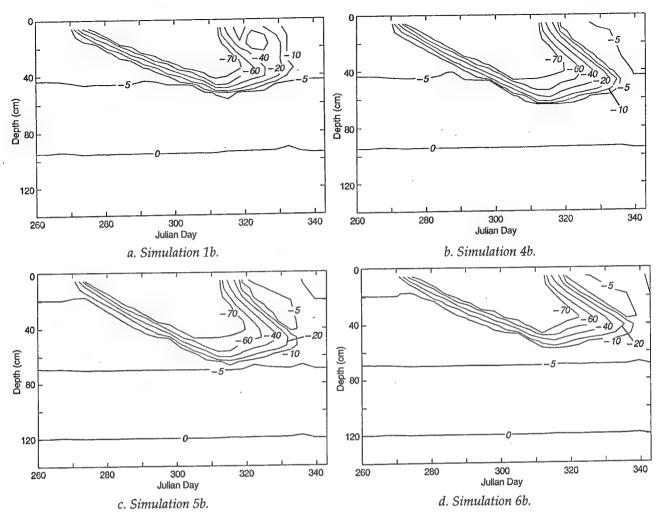


Figure 11. Predicted pore water pressures (kPa) for the wet case.

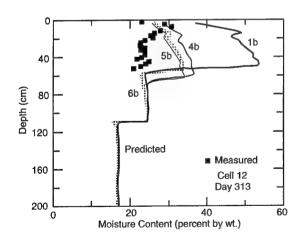


Figure 12. Predicted vs. measured gravimetric water content of the frozen core taken on the last day of the wet freeze event (Julian Day 313). The parameters for cases 1b, 4b, 5b, 6b are listed in Table 4.

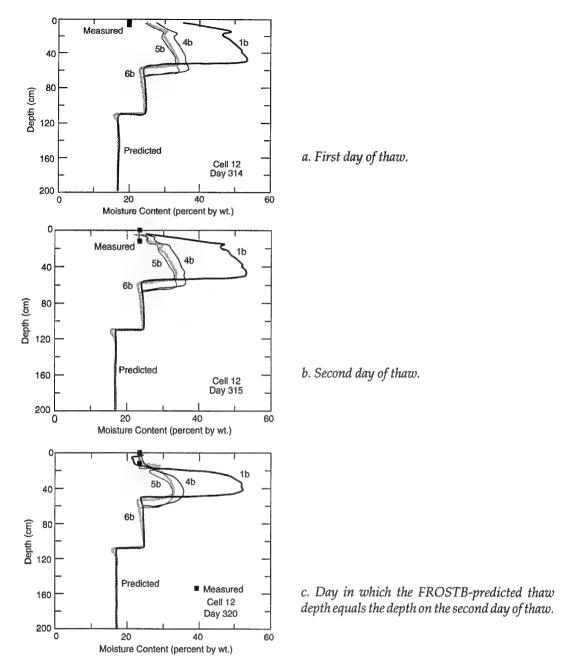


Figure 13. Predicted vs. measured gravimetric water content of the thawed soil samples from the wet freeze event.

of the results, are given in Table 5. All three cases started with the initial temperature conditions shown in Figure 14. Cases 8b and 9b started with the initial pore water pressure conditions set equal to the measured conditions from the surface down to the position of the deepest tensiometer (1.37 m) and then a constant gradient of 1 cm of water with each centimeter of depth down to the bottom, effectively placing a water table 1.67 m below the surface, a situation that was not present in the actual section (Fig. 14). This gradient and a constant

lower-boundary pore pressure were chosen so that the predicted pressures would equal the measured values (–30 cm of water) at the location of the deepest tensiometer. A third case (10b) used initial conditions with the measured pore water pressures down to 1.37 m and a constant water pressure of –30 cm of water from that point to the bottom, effectively moving the water table to a position 30 cm below the test section. Although this places the soil profile in a state of nonequilibrium, it more accurately simulates the experimental con-

Table 5. Dry case simulation summary. Cell 10; Initial temps from Julian Day 110; Upper boundary at 5.08-cm (2-in.) depth.

Case	Lower pressure	Water table (cm)	E-factor	Results
8b	Match bottom 167 Calc. tensiometer (8) (-30 cm of water)			Heave higher than measured Moisture high
9b	Match bottom tensiometer	167	Set (24)	Heave and max frost depth match Moisture high
10b	-30 cm of water at base	381	Set (24)	No heave Frost slightly too deep Moisture still high

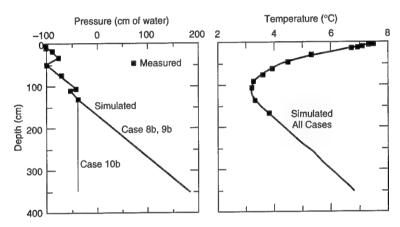


Figure 14. Initial conditions of temperature and pore water pressure (dry case, Julian Day 110) (10 cm of water = 1 kPa).

ditions at the beginning of freeze (i.e. gravity drainage of the soil column). In case 10b a constant lower-boundary pore pressure was set to maintain the water table at 30 cm below the test section throughout the simulation.

In the first case (8b) the E-factor was set to be calculated by the program, which produced a value of 8. As in the wet case the predicted values of heave were higher than measured, and the maximum predicted frost penetration was slightly less (Fig. 15a). The next simulation (9b) used a set value of E that had been "calibrated" earlier for the wet case. Its predictions (Fig 15b) gave heave amounts in the lower range of the measured data and a maximum frost penetration nearly equal to the measured depth. A third simulation (10b) used the set E-factor and lower-boundary pressures to simulate a water table about 30 cm below the bottom of the cell. This simulation predicted no heave and a maximum frost penetration deeper than the measured values (Fig. 15c).

Contour plots of predicted temperatures (Fig. 16) compare quite well with the measured data (Fig. 5a).

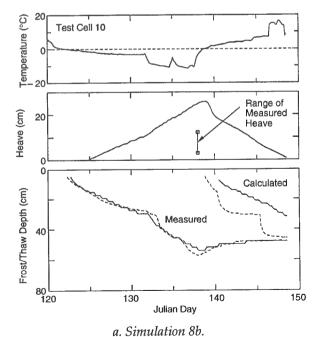


Figure 15. Applied upper-boundary temperatures, predicted frost heave and frost penetration compared to measured values for the dry case.

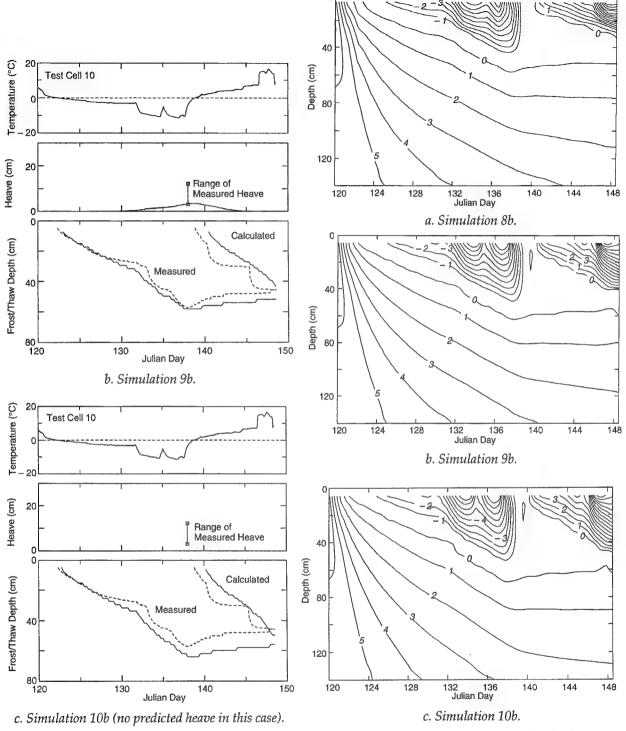


Figure 15 (cont'd). Figure 16. Predicted temperatures ($^{\circ}$ C) for the dry case.

As in the wet case the predicted pore water pressures (Fig. 17) are much higher than those measured (Fig. 5b).

Figures 18 and 19 compare the moisture predictions vs. the measured values for the dry cases. The predicted gravimetric water contents for the frozen

core are much higher than the measured values (Fig. 18). Predictions for the first day of thaw are extremely high as well (Fig. 19a), and even when FROSTB predicts a thaw depth equal to the first day of thaw, the predicted values are much higher than the measured values (Fig. 19b).

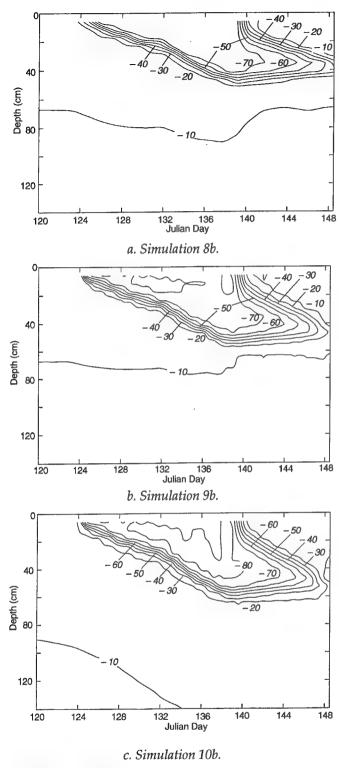


Figure 17. Predicted pore water pressures (kPa) for the dry case.

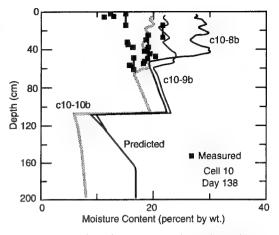
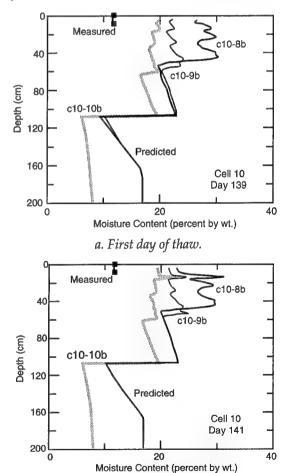


Figure 18. Predicted vs. measured gravimetric water content of the frozen core taken on the last day of the dry freeze event (Julian Day 138). The parameters for cases 8b, 9b and 10b are listed in Table 5.



b. Day in which the FROSTB-predicted thaw depth equals the depth on the first day of thaw.

Figure 19. Predicted vs. measured gravimetric water content of the thawed soil samples from the dry freeze event.

DISCUSSION

Overall the model predicts the frost penetration and heave quite well. However, it tends to overpredict the amount of ice formation. The additional ice in the frozen soil then causes a slower thaw than measured because of the time required to thaw the excess ice. One reason for the high predicted ice content is the assumption that the soil must be 100% saturated for frost heaving to occur. Moist soils will usually only saturate to 85–95% due to effective hydraulic conductivities and porosities (Dirksen and Miller 1966). Additional air entrapment may also occur during freezing, as air is included within ice crystals and ice lenses, particularly during rapid freezing. Based on closedsystem freezing of unsaturated soils, Dirksen and Miller (1966) suggested that frost heave occurs when the soil saturation reaches 90% rather than 100%. This is supported by the water content measurements from the wet case, which indicate an average saturation of the frozen soil of 87% (based on an average frozen density of 1.532 g/cm³). Therefore, modifying the model to account for a saturation of 90% would more closely simulate the freezing process and reflect the measured water contents and thaw progress.

For the dry case where a water table is not present yet measurable frost heave occurs (average heave $= 1.2 \, \text{cm}$), the measured total water content indicates an average saturation of only 40%. Simulation 9b generated frost heave within the range measured, but the model predicted an excess of ice formation. Simulation 10b, the vertical drainage case that more accurately simulates the initial soilwater conditions, had less ice formation but no frost heave. However, even simulation 10b predicted that the total water saturation of the frozen soil would be 75%, which is still considerably higher than the measured soil conditions. Since the predicted and measured soil densities are nearly the same, the reasons for overpredicting ice content and underpredicting frost heave could be:

- The laboratory-measured hydraulic properties of unfrozen soil and the field hydraulic properties of the freezing soil are different.
- The physical process of soil freezing and frost heave in unsaturated soil is not adequately understood.
- The flow potential of the liquid water should be based on a three-phase system (air-waterice) rather than a two-phase system (waterice).

Similar problems predicting soil moisture con-

ditions during freeze and thaw were reported by Xia Xu et al. (1991), although they underpredicted rather than overpredicted soil moisture. They suggested that the discrepancy is due to inaccurate measurements of soil hydraulic properties and the complications involved in predicting moisture movement in a multiphase system. Our simulation uses laboratory-measured flow properties based on the soil density in the experimental test bed: therefore, this is not likely to be a cause of error. However, there are problems associated with applying these properties based on unsaturated flow (an air-water-soil system) to a water-ice-soil system or a water--ice-air-soil system. The model currently accounts for changes in hydraulic conductivity as ice begins to form by reducing the hydraulic conductivity values using an E-factor (eq 4 and 5). This factor is empirically based and adjusts the hydraulic conductivity for changes in surface tensions and tortuosity as ice forms in soil pores (Guymon et al. 1993). Use of the provided function to calculate the E-factor produced heave much greater than measured in this study. Further calibration of the E-factor using frost susceptibility test data should be examined to determine whether another function should be utilized. Another solution would be to replace the E-factor with a function that varies pore water pressure with temperature, as has been implemented in the Integrated Model (Lytton et al. 1990).

Some improvement in the soil moisture predictions can also be made by adjusting the minimum unfrozen water content. For this study we used a value calculated by substituting a pore pressure equal to -800 cm of water in eq 2. Analysis of data from an unfrozen water content test on the Lebanon sand (App. A) indicates that a pore pressure on the order of -297 cm of water would be developed at a temperature of -10°C. When a simulation was run using a minimum unfrozen water content based on a pore pressure of -297 cm of water $(0.167 \text{ cm}^3/\text{m}^3)$ cm³), the predicted ice contents were reduced by about 10% relative to those predicted with a pore pressure of -800 cm of water, but they did not reach the measured values. Changing this value reduces the amount of water being drawn from underlying soil layers and produces less-negative pore pressures in the freezing zone.

Measurable frost heave in unsaturated soils was also observed in other freeze-thaw cycles produced in the FERF. Using the same moist soil and no water table, a separate freeze-thaw cycle (JD 160 to 201) resulted in an average frost heave of 1.7 cm for a frost depth of 43 cm and total water contents

in the frozen zone corresponding to an average saturation of 61.1%. However, small sections of the frozen soil core indicated total water contents of 88 and 89%, which would be enough to generate heaving according to our earlier arguments. Based on these measurements, it appears that the heaving of unsaturated soils in a closed system is isolated within the soil profile and that small lenses of nearly saturated soil and heave may lie adjacent to relatively dry layers (saturation of 41–62%). Some of this layering can be seen in the profile of total water contents of the frozen core sampled at 2.5-cm increments (Table 6). The water migration toward

Table 6. Total water content and saturation for the freeze cycle from June 8 to July 19 (JD 160 to 201).

Sample number	Water content (%)	Saturation (%)*	Sample number	Water content (%)	Saturation (%)*
I	12.51	45	11	20.15	73
2	11.69	42	12	17.50	63
3	11.84	43	13	17.25	62
4	12.3	45	14	24.54	89
5	12.45	45	15	24.21	88
6	13.74	50	16	18.73	68
7	12.31	45	17	11.28	41
8	16.45	60	18	21.54	78
9	19.65	71	19	17.83	65
10	21.68	78	20	19.48	71

^{*} Based on an average density of frozen soil of 1.55 g/cm³. Notes: No water table was present, the frost heave was 1.7 cm, the freeze depth was 50 cm and the sample thickness was 2.5 cm.

the freezing soil and the accompanying soil drying preceding the freezing front have been carefully documented in the laboratory by Dirksen and Miller (1966) and Nakano and Tice (1990). Dirksen also suggested that this process occurs on a microscopic scale and therefore is not always detected in our macroscale measurements.

Most freeze—thaw models assume that the soil becomes saturated as it begins to freeze, form ice lenses and heave, because this reduces the problem to a two-phase system. Even for moist soil with a nearby water supply, the soil is not likely to be 100% saturated, as discussed earlier, and therefore even this case is a three-phase system. Kung and Steenhuis (1986) attempted to model heat and moisture transfer in an unsaturated, partly frozen soil, but in doing so they assumed that the ice pressure is atmospheric, a good assumption if there is no frost heave. When modeling frost heave, however, Miller (1973) and Black (1991) supported the existence and importance of the pressure in the ice phase based on the geometry of the pore ice and

water as predicted theoretically by Miller (1973) and photographed by Colbeck (1982).

The FROSTB model currently calculates flow potential for the liquid water based on the soil moisture-tension curves measured in the laboratory and assumes that this potential is the same for either an air–water system (ϕ_{aw}) or an ice–water system (ϕ_{iw}). Experiments by Koopsman and Miller (1966, later expounded on by Miller 1973 and Black and Tice 1989) showed that this is the case for colloidal soils but that ϕ_{aw} and ϕ_{iw} are related by the ratio of the surface tensions ($\phi_{aw} = 2.2 \phi_{iw}$) in non-colloidal or capillary soils such as the granular Lebanon sand used in these experiments. Based on this the liquid flow potential for a three-phase system can be estimated by weighting the ϕ_{aw} and ϕ_{iw} potentials based on the percentage of liquid in contact with the air and water volumes, respectively:

$$\phi_{w} = \phi_{aw} \left(\frac{\theta_{a}}{\theta_{a} + \theta_{i}} \right) + \phi_{iw} \left(\frac{\theta_{i}}{\theta_{a} + \theta_{i}} \right)$$
 (9)

where ϕ_w = liquid water flow potential for a threephase system

 ϕ_{aw} = flow potential in an air-water-soil system

 ϕ_{iw} = flow potential in an ice–water–soil system = $\phi_{aw}/2.2$

 θ_a = volumetric air content

 θ_i = volumetric ice content.

This "pseudo" three-phase potential effectively reduces the liquid water flow potential in the freezing soil and would therefore reduce the flow volume and the resulting total water contents in the frozen zone. An added benefit of this modification to the flow potential is that the controversial *E*-factor may have less importance.

CONCLUSIONS

The results of the simulations show that FROSTB predicts ice contents higher than measured in the FERF tests. An effect of the high ice predictions is a delay in thawing, because it takes longer to thaw the excess ice. Predictions of thawed moisture contents were close to those measured in the wet case, once FROSTB had predicted a thaw depth equal to measured thaw depths. In the dry case the predicted moisture contents were always higher than measured, even in case 10b, where a deep water table was simulated and moisture predictions from equivalent thaw depths were compared with the measured data.

Experimental studies indicate that the process of freezing and frost heaving in unsaturated soils (with no water supply) occurs by heaving of nearly saturated layers adjacent to relatively dry soil layers and that this layering can occur on a microscopic level. Total water content measurements of the frozen soil also indicate water contents less than 90% of saturation in many cases. A possible solution to reducing the amount of predicted ice would be to allow ice formation prior to saturation of an element, that is, incorporate some air in frozen elements by triggering frost heave at 90% saturation. In addition, modifying the flow potential to account for the presence of three separate phases (eq 9) effectively reduces the potential gradient for flow into the frozen soil. The flow potential could also be reduced by increasing the minimum volumetric water content, effectively reducing the negative pore pressures in the freezing zone.

Another possible solution for reducing the prediction of excess ice would be a more rigorous evaluation of the *E*-factor function using frost-susceptibility test data or replacement of the *E*-factor with a function that varies pore water pressure with temperature.

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APPENDIX A: UNFROZEN WATER CONTENT DATA FOR LEBANON SAND

Table A1. Warming data from unfrozen water content test on Lebanon sand. Four samples tested with thawed gravimetric water contents are shown.

Temp (°C)	Unfrozen water content (%), gravimetric					
>0	5.52	10.87	15.74	20.65		
-3.00	0.32		0.2	0.18		
-1.93	0.63	0.41	0.39	0.37		
-1.46	0.95	0.62	0.59	0.55		
-0.96	1.26	0.82	0.79	0.74		
-0.78	1.26	1.03	0.99	0.74		
-0.59	1.58	1.03	0.99	0.92		
-0.48	1.89	1.24	1.18	1.29		
-0.37	1.89	1.65	1.38	1.29		
-0.29	2.2	1.65	1.58	1.29		
-0.17	2.52	2.06	1.78	1.67		
-0.08	4.09	2.89	2.57	2.04		
-0.04	4.72	3.51	2.96	2.59		
-0.01		7.64	6.32	5.93		

Table A2. Coefficients for modeling unfrozen water content of Lebanon sand using the relationship of Tice et al. (1982): $W_u = \alpha \ |T|^{\beta}$, where W_u is gravimetric unfrozen water content, T is temperature in ${}^{\circ}C$, and α and β are coefficients.

Sample water content	Coeff	icients
(%)	α	β
5.52	1.0045	-0.5629
10.87	0.7949	-0.5069
15.74	0.6685	-0.5297
20.65	0.6075	-0.5235

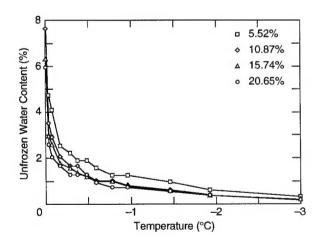


Figure A1. Warming data from an unfrozen water content test on Lebanon sand. Four samples tested with thawed gravimetric water contents are shown.

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